15.401 Finance Theory

MIT Sloan MBA Program

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Lectures 4–6: Fixed-Income Securities

Critical Concepts

- Industry Overview
- Valuation
- Valuation of Discount Bonds
- Valuation of Coupon Bonds
- Measures of Interest-Rate Risk
- Corporate Bonds and Default Risk
- The Sub-Prime Crisis

Readings

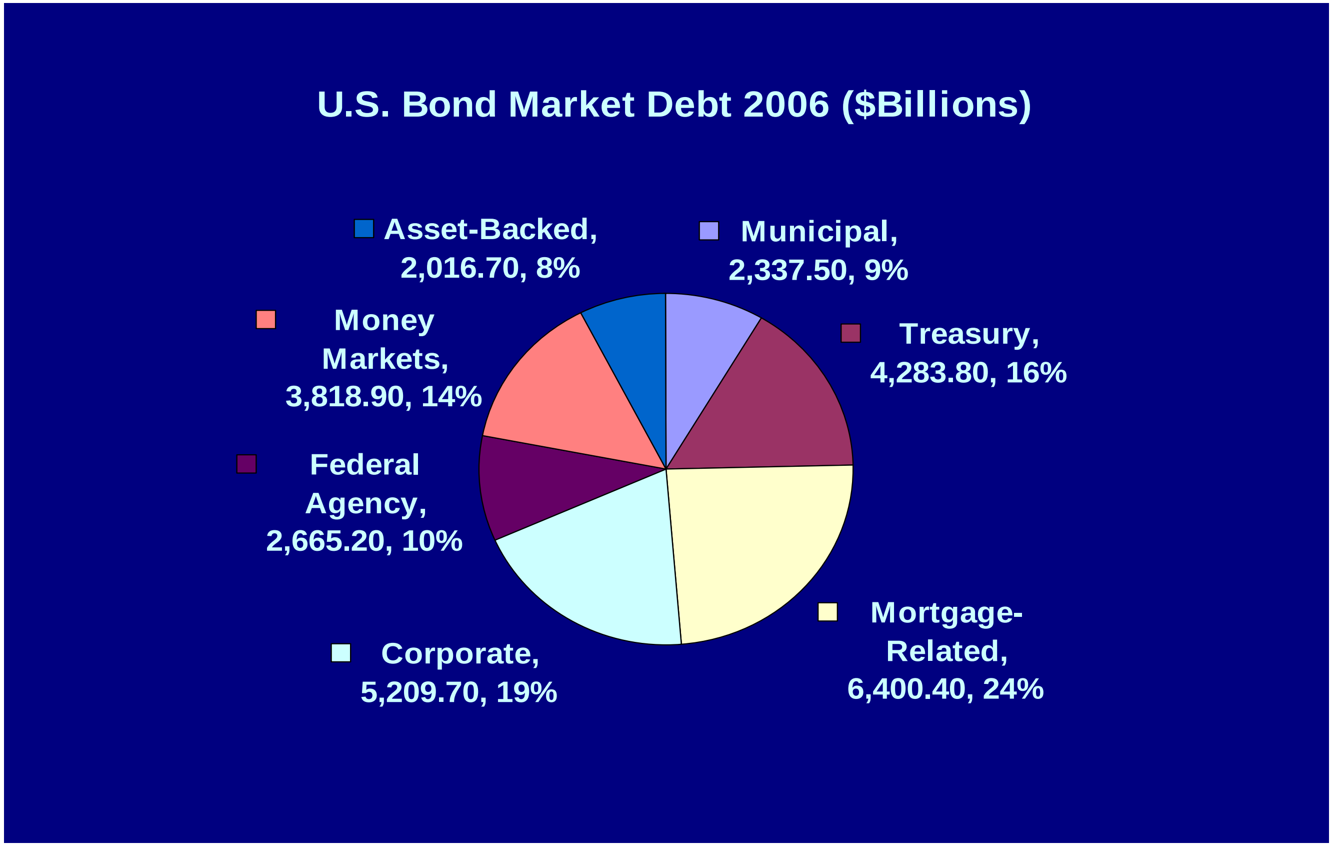
- Brealey, Myers, and Allen Chapters 23–25

Fixed-income securities are financial claims with promised cashflows of known fixed amount paid at fixed dates.

Classification of Fixed-Income Securities:

- Treasury Securities
  - U.S. Treasury securities (bills, notes, bonds)
  - Bunds, JGBs, U.K. Gilts
  - ….
- Federal Agency Securities
  - Securities issued by federal agencies (FHLB, FNMA $\ldots$)
- Corporate Securities
  - Commercial paper
  - Medium-term notes (MTNs)
  - Corporate bonds
  - ….
- Municipal Securities
- Mortgage-Backed Securities
- Derivatives (CDO’s, CDS’s, etc.)

Industry Overview



Industry Overview

Outstanding U.S. Bond Market Debt

$ Billions

sifma

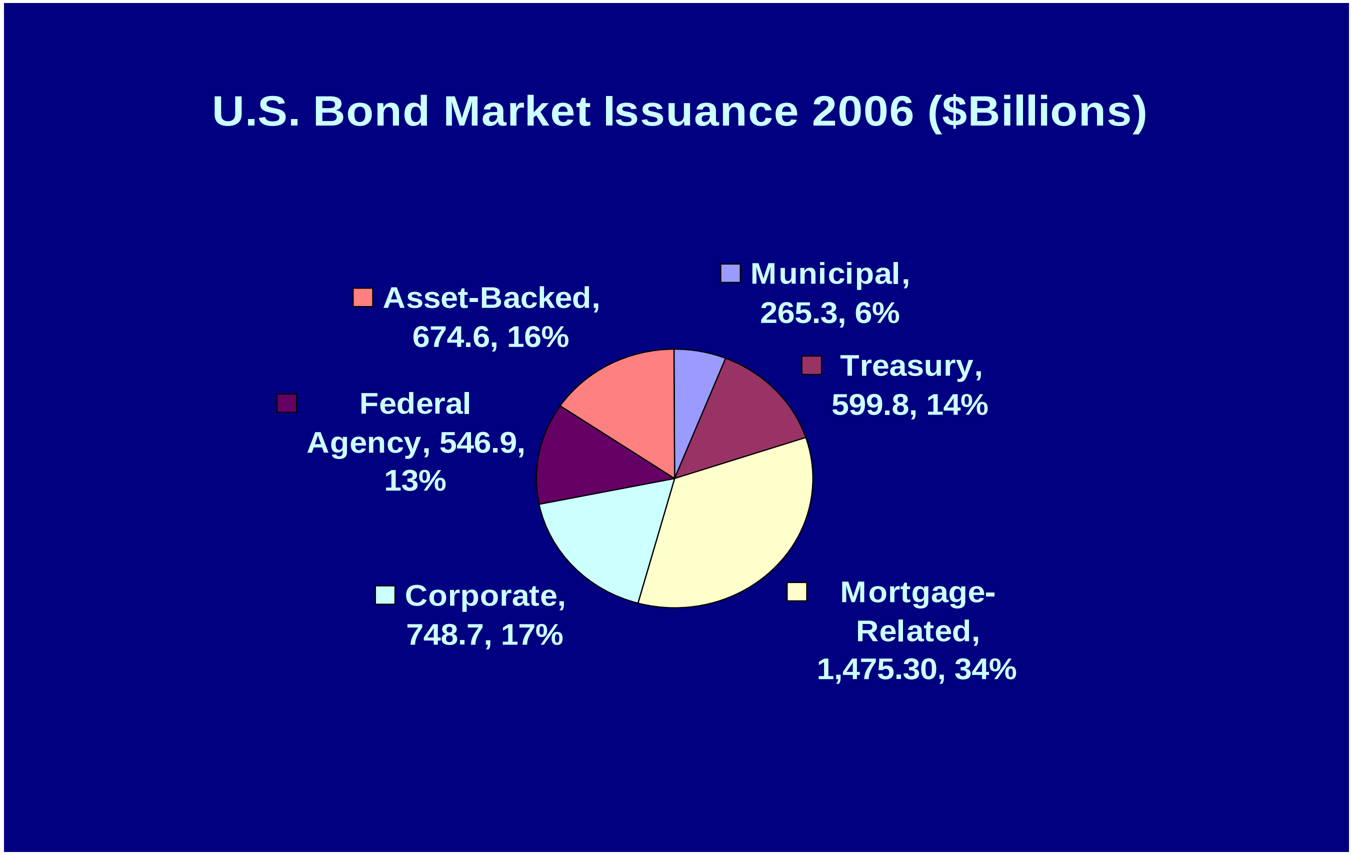
Invested in America

	Municipal	Treasury[1]	Mortgage Related[2]	Corporate Debt	Federal Agency Securities[5]	Money Markets[3]	Asset-Backed[4]	Total
1996	1,261.6	3,666.7	2,486.1	2,126.5	925.8	1,393.9	365.4	12,226.0
1997	1,318.7	3,659.5	2,680.2	2,359.0	1,021.8	1,692.8	535.8	13,267.8
1998	1,402.7	3,542.8	2,955.2	2,708.5	1,302.1	1,977.8	731.5	14,620.6
1999	1,457.1	3,529.5	3,334.3	3,046.5	1,620.0	2,338.8	900.8	16,227.0
2000	1,480.5	3,210.0	3,565.8	3,358.4	1,853.7	2,662.6	1,071.8	17,202.8
2001	1,603.6	3,196.6	4,127.4	3,836.4	2,157.4	2,587.2	1,281.2	18,789.8
2002	1,763.0	3,469.2	4,686.4	4,132.8	2,377.7	2,545.7	1,543.2	20,518.0
2003	1,900.7	3,967.8	5,238.6	4,486.5	2,626.2	2,519.8	1,693.7	22,433.3
2004	2,030.9	4,407.4	5,930.5	4,801.8	2,700.6	2,904.2	1,827.8	24,603.2
2005	2,226.0	4,714.8	7,212.3	4,965.7	2,616.0	3,433.7	1,955.2	27,123.7
2006	2,403.4	4,872.4	8,635.4	5,344.6	2,651.3	4,008.8	2,130.4	30,046.3
2007	2,618.9	5,075.4	9,142.7	5,946.8	2,933.3	4,171.3	2,472.4	32,360.8
2008	2,680.4	6,082.5	9,101.9	6,201.6	3,210.5	3,790.9	2,671.8	33,739.6
2009	2,811.2	7,610.3	9,187.7	6,869.0	2,727.3	3,127.8	2,429.0	34,762.3

1 Interest bearing marketable public debt.
2 Includes GNMA, FNMA, and FHLMC mortgage-backed securities and CMOs, and CMBS, and private-label MBS/CMOs.
3 Includes commercial paper, bankers acceptances, and large time deposits.
4 Includes auto, credit card, home equity, manufacturing, student loans and other; CDOs of ABS are included
5 Due to FAS 166/167 changes, the GSE debt category in the Federal Reserve is no longer our source for agency debt going forward from Q1 2010. Contains agency debt of Fannie Mae, Freddie Mac, Farmer Mac, FHLB, the Farm Credit System, and federal budget agencies (e.g., TVA)

Sources: U.S. Department of Treasury, Federal Reserve System, Federal agencies, Dealogic, Thomson Reuters, Bloomberg, Loan Performance and SIFMA

Industry Overview



Industry Overview

Issuance in the U.S. Bond Markets
USD Billions

sifma
Invested in America

	Municipal	Treasury[1]	Mortgage-Related[2]	Corporate Debt[3]	Federal Agency Securities	Asset-Backed	Total
1996	185.2	612.4	492.6	343.7	277.9	168.4	2,080.2
1997	220.7	540.0	604.4	466.0	323.1	223.1	2,377.3
1998	286.8	438.4	1,143.9	610.7	596.4	286.6	3,362.7
1999	227.5	364.6	1,025.4	629.2	548.0	287.1	3,081.8
2000	200.8	312.4	684.4	587.5	446.6	281.5	2,513.2
2001	287.7	380.7	1,671.3	776.1	941.0	326.2	4,383.0
2002	357.5	571.6	2,249.2	636.7	1,041.5	373.9	5,230.4
2003	382.7	745.2	3,071.1	775.8	1,267.5	461.5	6,703.8
2004	359.8	853.3	1,779.0	780.7	881.8[(4)]	651.5	4,424.3
2005	408.2	746.2	1,966.7	752.8	669.0	753.5	5,296.4
2006	386.5	788.5	1,987.8	1,058.9	747.3	753.9	5,722.9
2007	429.3	752.3	2,050.3	1,127.5	941.8	509.7	5,810.9
2008	389.5	1,037.3	1,344.1	707.2	984.5	139.5	4,602.1
2009	409.6	2,185.5	1,957.2	901.8	1,117.0	146.2	6,717.2

[1] Interest bearing marketable coupon public debt.
[2] Includes GNMA, FNMA, and FHLMC mortgage-backed securities and CMOs and private-label MBS/CMOs.
[3] Includes all non-convertible debt, MTNs and Yankee bonds, but excludes CDs and federal agency debt.
[4] Beginning with 2004, Sallie Mae has been excluded due to privatization.

Sources: U.S. Department of Treasury, Federal Agencies, Thomson Reuters.

Industry Overview

U.S. Bond Markets
Average Daily Trading Volume
USD Billions

	Municipal	Treasury[1]	Agency MBS[1]	Corporate Debt[2]	Federal Agency Securities[1]	Total[3]
1996	1.1	203.7	38.1	-	31.1	274.0
1997	1.1	212.1	47.1	-	40.2	300.5
1998	3.3	226.6	70.9	-	47.6	348.5
1999	8.3	186.5	67.1	-	54.5	316.5
2000	8.8	206.5	69.5	-	72.8	357.6
2001	8.8	297.9	112.0	-	90.2	508.9
2002	10.7	366.4	154.5	16.3	81.8	629.7
2003	12.6	433.5	206.0	18.0	81.7	751.8
2004	14.8	499.0	207.4	18.8	78.8	818.9
2005	16.9	554.5	251.8	16.7	78.8	918.7
2006	22.5	524.7	254.6	16.9	74.4	893.1
2007	25.1	570.2	320.1	16.4	83.0	1,014.9
2008	19.4	553.1	344.9	11.8	104.5	1,033.6
2009	12.5	407.9	299.9	16.8	77.7	814.6

[1] Primary dealer activity

[2] Excludes all issues with maturities of one year or less and convertible securities

[3] Totals may not add due to rounding

Sources: Federal Reserve Bank of New York, Municipal Securities Rulemaking Board, FINRA

Fixed-Income Market Participants

Issuers:
- Governments
- Corporations
- Commercial Banks
- States
- Municipalities
- SPVs
- Foreign Institutions

Intermediaries:
- Primary Dealers
- Other Dealers
- Investment Banks
- Credit-rating Agencies
- Credit Enhancers
- Liquidity Enhancers

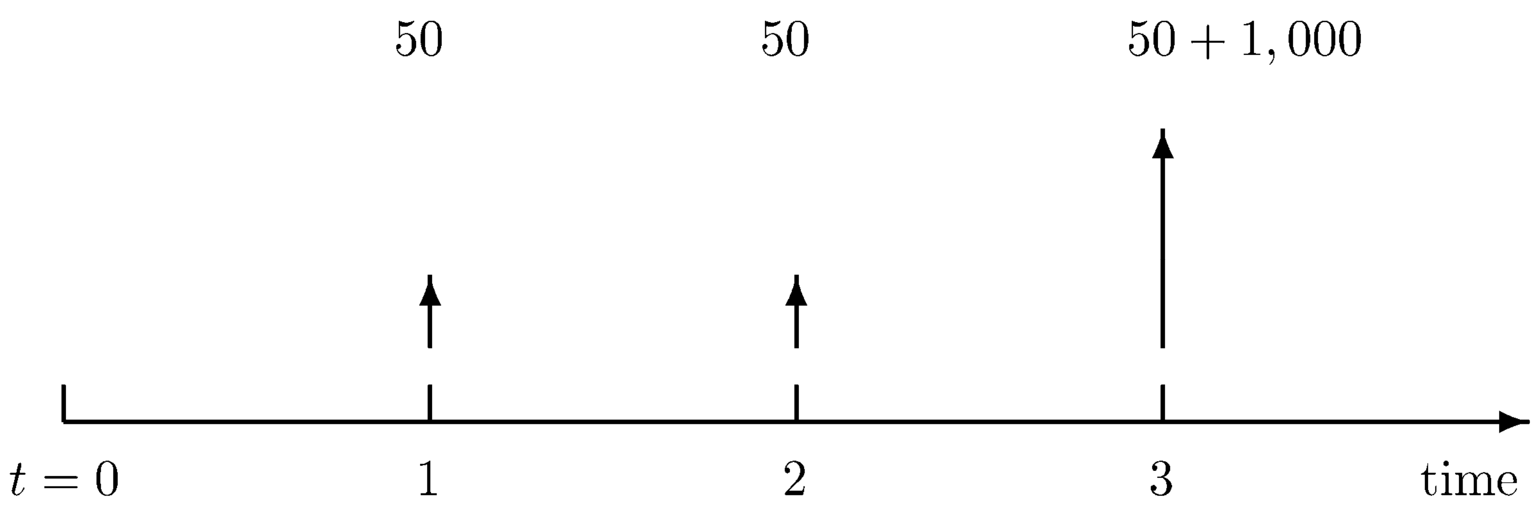
Investors:
- Governments
- Pension Funds
- Insurance Companies
- Commercial Banks
- Mutual Funds
- Hedge Funds
- Foreign Institutions
- Individuals

Valuation

Cashflow:

- Maturity
- Coupon
- Principal

Example. A 3-year bond with principal of $1,000 and annual coupon payment of 5% has the following cashflow:



Valuation

Components of Valuation

- Time value of principal and coupons
- Risks
  - Inflation
  - Credit
  - Timing (callability)
  - Liquidity
  - Currency

For Now, Consider Riskless Debt Only

- U.S. government debt (is it truly riskless?)
- Consider risky debt later

Valuation of Discount Bonds

Pure Discount Bond

- No coupons, single payment of principal at maturity
- Bond trades at a “discount” to **face value**
- Also known as **zero-coupon bonds** or **STRIPS***
- Valuation is straightforward application of NPV

$$P_0 = \frac{F}{(1+r)^T}$$

- Note: (P_0, r, F) is “over-determined”; given two, the third is determined

Now What If r Varies Over Time?

- Different interest rates from one year to the next
- Denote by r_t the **spot rate of interest** in year t

***S**eparate **T**rading of **R**egistered **I**nterest and **P**rincipal **S**ecurities

If r Varies Over Time

- Denote by R_t the **one-year spot rate of interest** in year t

$$P_0 = \frac{F}{(1+R_1)(1+R_2)\cdots(1+R_T)}$$

- But we don't observe the entire sequence of future spot rates today!

$$\begin{aligned} P_0 &= \frac{F}{(1+R_1)(1+R_2)\cdots(1+R_T)} \\ &= \frac{F}{(1+r_{0,T})^T} \quad , \quad r_{0,T} \equiv \text{Today's } T\text{-Year Spot Rate} \end{aligned}$$

- Today's **T-year spot rate is** an "average" of one-year future spot rates
- $(P_0, F, r_{0,T})$ is over-determined

Valuation of Discount Bonds

Example:

On 20010801, STRIPS are traded at the following prices:

Maturity (year)	1/4	1/2	1	2	5	10	30
Price	0.991	0.983	0.967	0.927	0.797	0.605	0.187

For the 5-year STRIPS, we have

$$0.797 = \frac{1}{(1+r_{0,5})^5} \Rightarrow r_{0,5} = \frac{1}{(0.797)^{1/5}} - 1 = 4.64\%$$

Suppose We Observe Several Discount Bond Prices Today

$$P_{0,1} = \frac{F}{(1+R_1)} \quad \longrightarrow \quad r_{0,1}$$

$$P_{0,2} = \frac{F}{(1+R_1)(1+R_2)} \quad \longrightarrow \quad r_{0,2}$$

$$P_{0,3} = \frac{F}{(1+R_1)(1+R_2)(1+R_3)} \quad \longrightarrow \quad r_{0,3}$$

$$\vdots$$

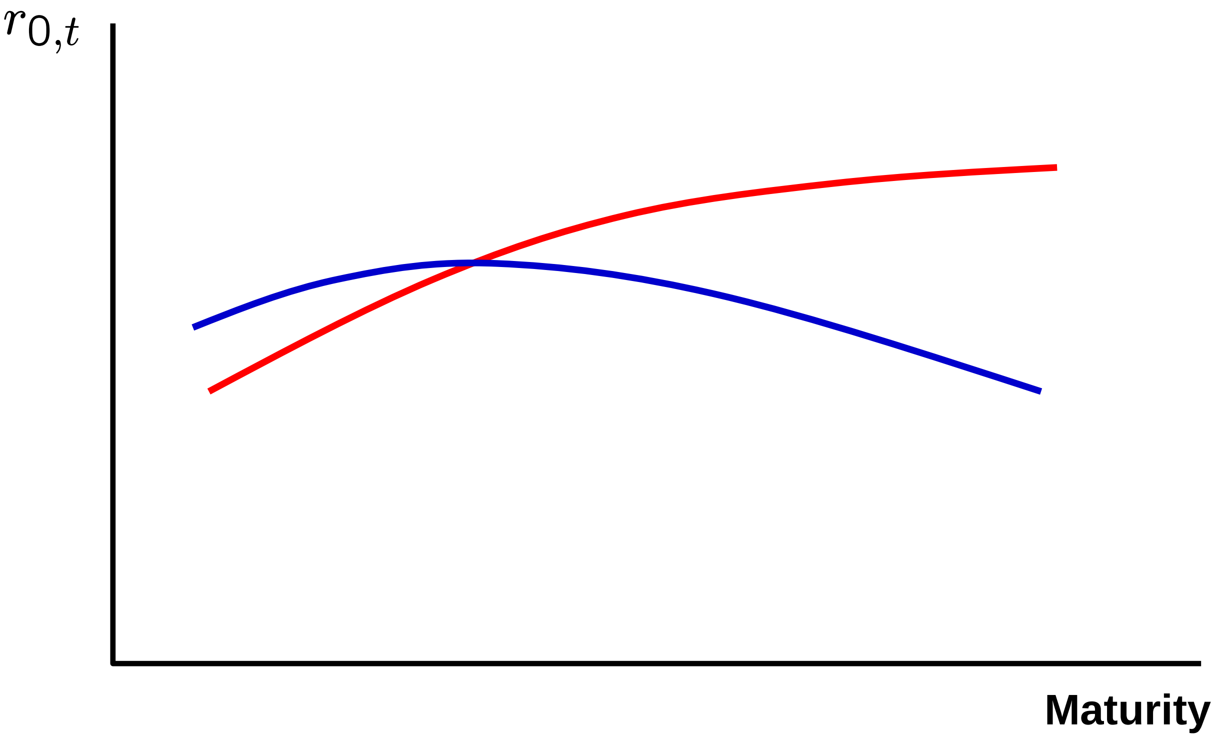
$$P_{0,T} = \frac{F}{(1+R_1)(1+R_2)(1+R_3)\cdots(1+R_T)} \quad \longrightarrow \quad r_{0,T}$$

$$\{P_{0,1}, P_{0,2}, \ldots, P_{0,T}\} \quad \longrightarrow \quad \{r_{0,1}, r_{0,2}, \ldots, r_{0,T}\}$$

Term Structure of Interest Rates

Valuation of Discount Bonds

Term Structure Contain Information About Future Interest Rates



- What are the implications of each of the two term structures?

Term Structure Contain Information About Future Interest Rates

$$P_{0,1} = \frac{F}{(1+R_1)}$$

$$P_{0,2} = \frac{F}{(1+R_1)(1+R_2)}$$

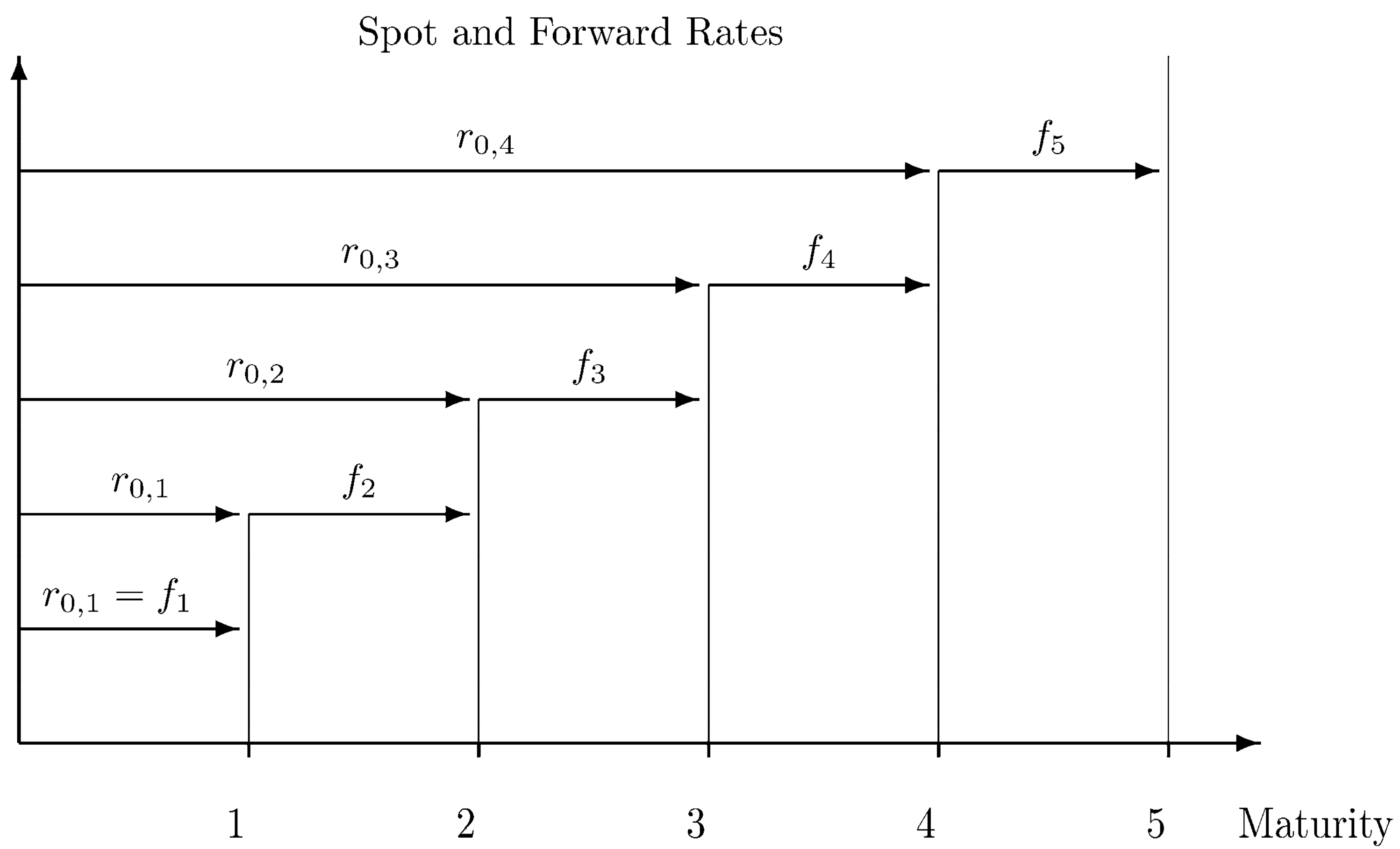
$$\frac{P_{0,1}}{P_{0,2}} = \frac{F}{(1+R_1)} \frac{(1+R_1)(1+R_2)}{F} = (1+R_2)$$

- Implicit in current bond prices are forecasts of future spot rates!
- These current forecasts are called **one-year forward rates**
- To distinguish them from spot rates, we use new notation:

$$\frac{P_{0,t-1}}{P_{0,t}} = 1 + f_t = \frac{(1+r_{o,t})^t}{(1+r_{o,t-1})^{t-1}}$$

Valuation of Discount Bonds

Term Structure Contain Information About Future Interest Rates



Valuation of Discount Bonds

More Generally:

- **Forward interest rates** are today's rates for transactions between two future dates, for instance, t_1 and t_2.
- For a forward transaction to borrow money in the future:
  - Terms of transaction is agreed on today, $t = 0$
  - Loan is received on a future date t_1
  - Repayment of the loan occurs on date t_2
- Note: future spot rates can be (and usually are) different from current corresponding forward rates

Valuation of Discount Bonds

Example:

As the CFO of a U.S. multinational, you expect to repatriate $10MM from a foreign subsidiary in one year, which will be used to pay dividends one year afterwards. Not knowing the interest rates in one year, you would like to lock into a lending rate one year from now for a period of one year. What should you do? The current interest rates are:

t	1	2
$r_{0,t}$	0.05	0.07

Strategy:

- Borrow $9.524MM now for one year at 5%
- Invest the proceeds $9.524MM for two years at 7%

Valuation of Discount Bonds

Example (cont):

Outcome (in millions of dollars):

Position	Year 0	Year 1	Year 2
1-Yr Borrowing	9.524	−10.000	0
2-Yr Lending	−9.524	0	10.904
Repatriation	0	10.000	0
Net	0	0	10.904

- The locked-in 1-year lending rate one year from now is 9.04%, which is the **one-year forward rate** for Year 2

Valuation of Discount Bonds

Example:

Suppose that discount bond prices are as follows:

t	1	2	3	4
P_t	0.9524	0.8900	0.8278	0.7629
$r_{0,t}$	0.05	0.06	0.065	0.07

A customer would like to have a forward contract to borrow $20MM three years from now for one year. Can you (a bank) quote a rate for this forward loan?

All you need is the forward rate f_4 which should be your quote for the forward loan

$$f_4 = \frac{(1+r_{0,4})^4}{(1+r_{0,3})^3} - 1 = \frac{(1.07)^4}{(1.065)^3} - 1 = 8.51\%$$

Valuation of Discount Bonds

Example (cont):

Strategy:

- Buy 20,000,000 of 3 year discount bonds, costing

$$(20,000,000)(0.8278) \quad = \quad \$16,556,000$$

- Finance this by **(short)selling** 4 year discount bonds of amount

$$16,556,000/0.7629 \quad = \quad \$21,701,403$$

- This creates a liability in year 4 in the amount $21,701,403
- Aside: A **shortsales** is a particular financial transaction in which an individual can sell a security that s/he does not own by borrowing the security from another party, selling it and receiving the proceeds, and then buying back the security and returning it to the orginal owner at a later date, possibly with a capital gain or loss.

Valuation of Discount Bonds

Example (cont):

- Cashflows from this strategy (in million dollars):

Position	Year 0	Years 1–2	Year 3	Year 4
Long 3-Year Bond	−16.556	0	20.000	0
Short 4-Year Bond	16.556	0	0	−21.701
Total	0	0	20.000	−21.701

- The yield for this strategy or “synthetic bond return” is given by:

$$\frac{21,701,403}{20,000,000} - 1 = 8.51\%$$

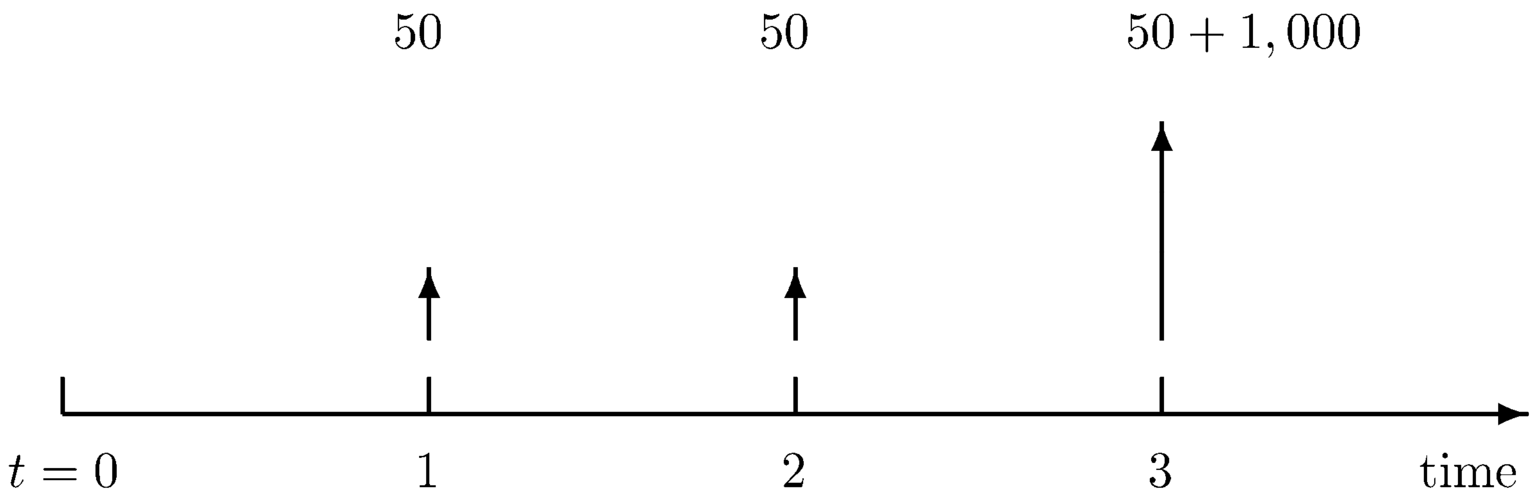
Valuation of Coupon Bonds

Coupon Bonds

- Intermediate payments in addition to final principal payment
- Coupon bonds can trade at discounts or premiums to face value
- Valuation is straightforward application of NPV (how?)

Example:

- 3-year bond of $1,000 par value with 5% coupon



Valuation of Coupon Bonds

$$P_0 = \frac{C}{(1+R_1)} + \frac{C}{(1+R_1)(1+R_2)} + \cdots + \frac{C+F}{(1+R_1)\cdots(1+R_T)}$$

- Since future spot rates are unobservable, summarize them with *y*

$$P_0 = \frac{C}{(1+y)} + \frac{C}{(1+y)^2} + \cdots + \frac{C+F}{(1+y)^T}$$

- *y* is called the **yield-to-maturity** of a bond
- It is a complex average of all future spot rates
- There is usually no closed-form solution for *y*; numerical methods must be used to compute it (T^{th}-degree polynomial)
- (P_0, *y*, *C*) is over-determined; any two determines the third
- For pure discount bonds, the YTM's are the current spot rates
- Graph of coupon-bond *y* against maturities is called the **yield curve**

Valuation of Coupon Bonds

U.S. Treasury Yield Curves

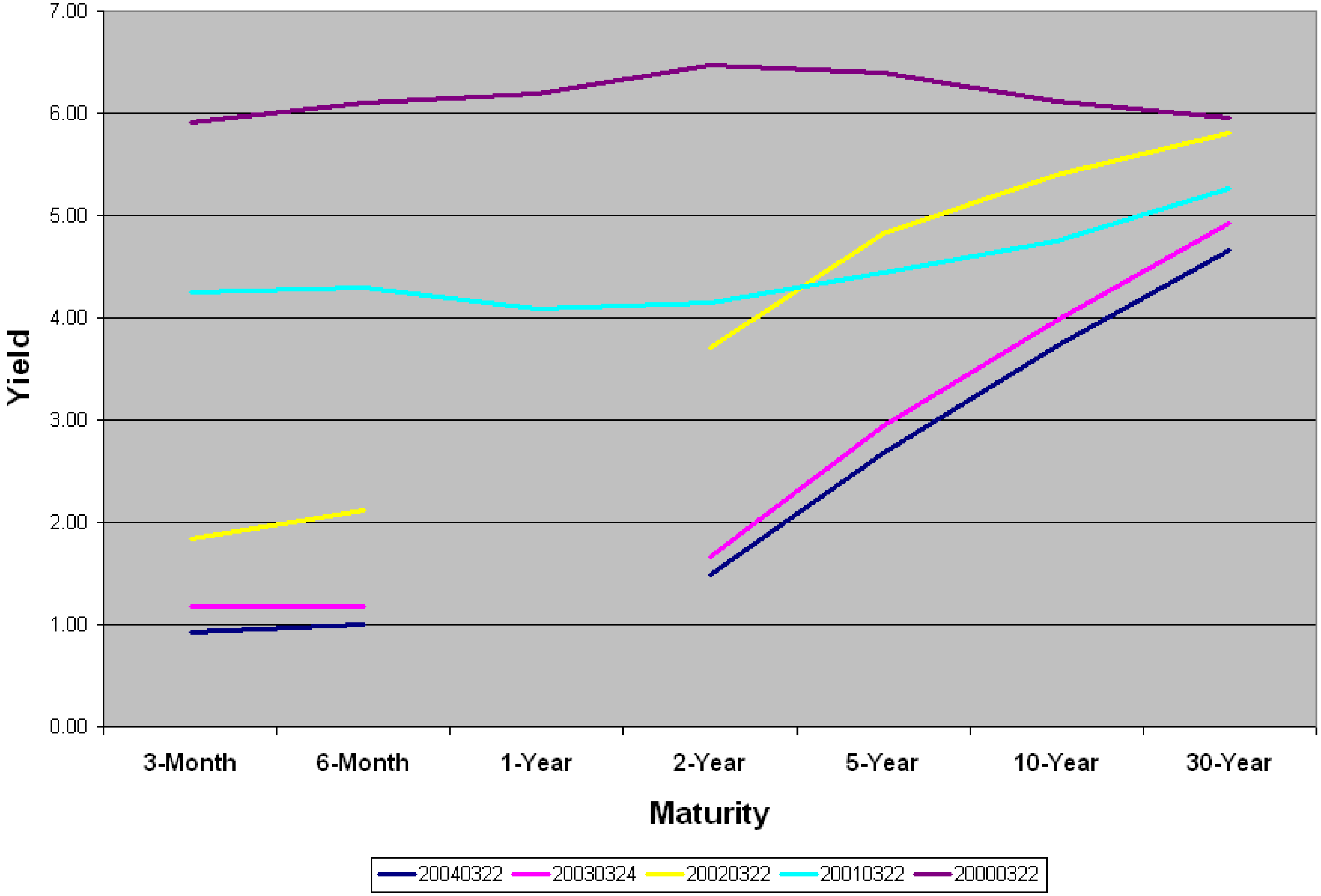
Statistic	Maturity (Years)						
	.25	.5	1	2	5	10	30
Start	19830601	19830601	19830601	19770131	19620105	19620105	19801201
End	20040322	20040322	20010831	20040322	20040322	20040322	20040322
Mean	5.36	5.56	6.35	7.24	7.02	7.24	8.02
SD	2.29	2.37	1.97	3.17	2.61	2.50	2.49
Max	0.81	0.81	2.96	1.08	2.03	3.11	4.17
Median	5.31	5.51	5.95	6.62	6.64	6.88	7.50
Min	11.15	11.59	12.31	16.96	16.27	15.84	15.21
Skewness	0.00	0.04	0.67	0.58	0.98	0.98	0.88
Kurtosis	−0.31	−0.22	0.14	0.21	0.95	0.71	−0.08
ρ_1	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20040322	0.93	1.00	—	1.48	2.68	3.72	4.67
20030324	1.17	1.18	—	1.66	2.94	3.97	4.93
20020322	1.84	2.12	—	3.71	4.82	5.40	5.81
20010322	4.25	4.29	4.10	4.15	4.45	4.76	5.27
20000322	5.92	6.10	6.19	6.48	6.39	6.11	5.96

Source: Bloomberg

Valuation of Coupon Bonds

Time Series of U.S. Treasury Security Yields

Valuation of Coupon Bonds



Models of the Term Structure

- Expectations Hypothesis
- Liquidity Preference
- Preferred Habitat
- Market Segmentation
- Continuous-Time Models
  – Vasicek, Cox-Ingersoll-Ross, Heath-Jarrow-Morton

Expectations Hypothesis

- Expected Future Spot = Current Forward

$$\mathrm{E}_0[R_k] \;=\; f_k$$

Valuation of Coupon Bonds

Liquidity Preference Model

- Investors prefer liquidity
- Long-term borrowing requires a premium
- Expected future spot < current forward

$$\mathrm{E}[R_k] \quad < \quad f_k$$

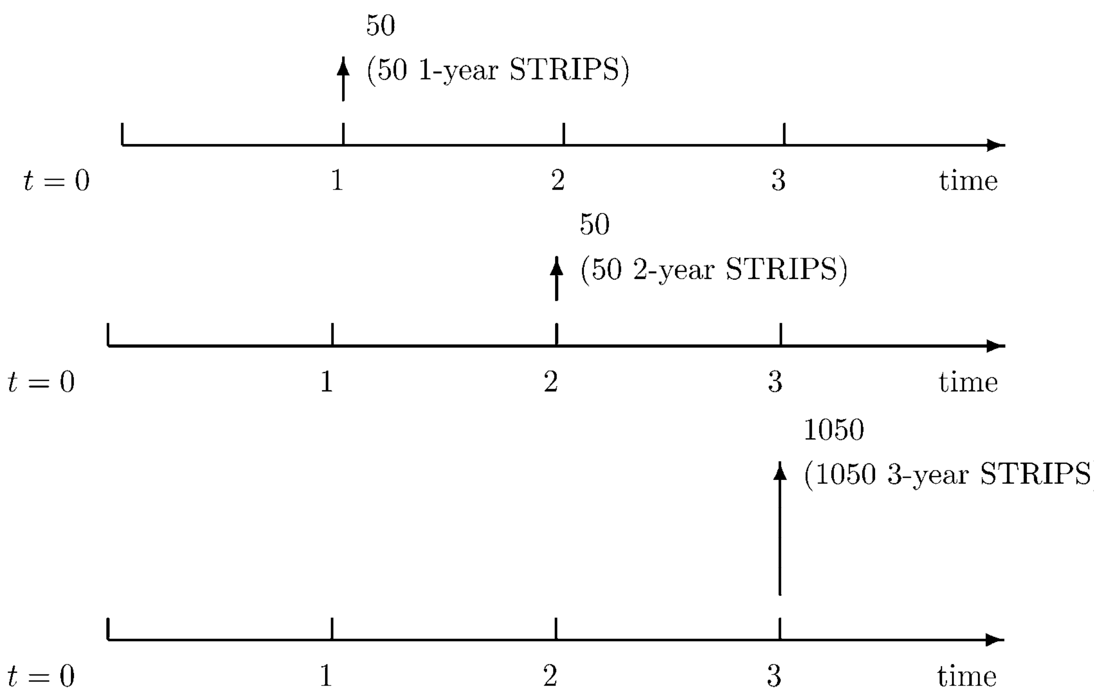
$$\mathrm{E}[R_k] \quad = \quad f_k - \text{Liquidity Premium}$$

Another Valuation Method for Coupon Bonds

- Theorem: All coupon bonds are portfolios of pure discount bonds
- Valuation of discount bonds implies valuation of coupon bonds
- Proof?

Example:

- 3-Year 5% bond
- Sum of the following discount bonds:
  - 50 1-Year STRIPS
  - 50 2-Year STRIPS
  - 1050 3-Year STRIPS



Valuation of Coupon Bonds

Example (cont):

- Price of 3-Year coupon bond must equal the cost of this portfolio
- What if it does not?

In General:

$$P \;=\; C\,P_{0,1} \;+\; C\,P_{0,2} \;+\; \cdots \;+\; (C+F)P_{O,T}$$

- If this relation is violated, **arbitrage opportunities** exist
- For example, suppose that

$$P \;>\; C\,P_{0,1} \;+\; C\,P_{0,2} \;+\; \cdots \;+\; (C+F)P_{O,T}$$

- Short the coupon bond, buy *C* discount bonds of all maturities up to *T* and F discount bonds of maturity *T*
- No risk, positive profits $\Rightarrow$ arbitrage

What About Multiple Coupon Bonds?

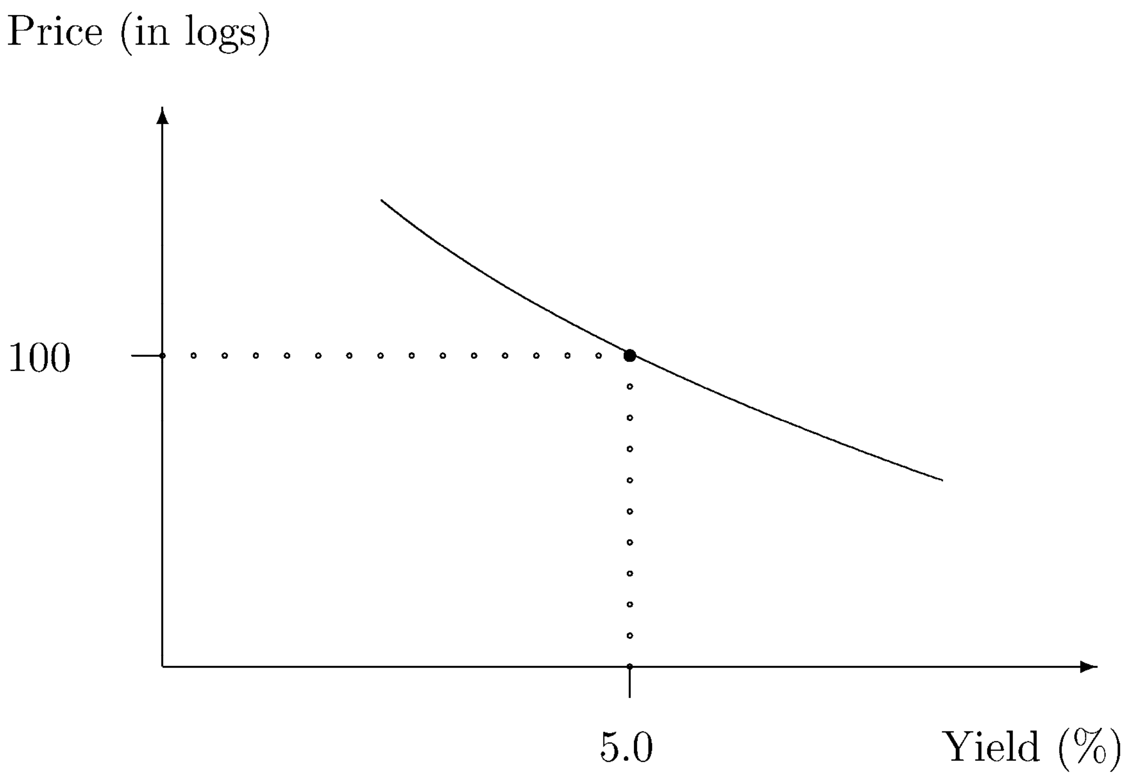
$$
\begin{aligned}
P_1 &= C_{11}\,P_{0,1} + C_{12}\,P_{0,2} + \cdots + C_{1T}\,P_{0,T} \\
P_2 &= C_{21}\,P_{0,1} + C_{22}\,P_{0,2} + \cdots + C_{2T}\,P_{0,T} \\
&\vdots \\
P_n &= C_{n1}\,P_{0,1} + C_{n2}\,P_{0,2} + \cdots + C_{nT}\,P_{0,T}
\end{aligned}
$$

- Suppose n is much bigger than T (more bonds than maturity dates)
- This system is over-determined: *T* unknowns, *n* linear equations
- What happens if a solution does not exist?
- This is the basis for **fixed-income arbitrage** strategies

Measures of Interest-Rate Risk

Bonds Subject To Interest-Rate Risk

- As interest rates change, bond prices also change
- Sensitivity of price to changes in yield measures risk



Macaulay Duration

- Weighted average term to maturity

$$D_m = \sum_{k=1}^{T} k \cdot \omega_k \qquad \sum_{k=1}^{q} \omega_k = 1$$

$$\omega_k = \frac{C_k/(1+y)^k}{P} = \frac{\text{PV}(C_k)}{P}$$

- Sensitivity of bond prices to yield changes

$$P = \sum_{k=1}^{T} \frac{C_k}{(1+y)^k}$$

$$\frac{\partial P}{\partial y} = \frac{-1}{1+y} \sum_{k=1}^{T} k \cdot \frac{C_k}{(1+y)^k}$$

$$\frac{1}{P}\frac{\partial P}{\partial y} = -\frac{D_m}{1+y}$$

$$= -D_m^* \qquad \text{Modified Duration}$$

Example:

Consider a 4-year T-note with face value \$100 and 7% coupon, selling at \$103.50, yielding 6%.

- For T-notes, coupons are paid semi-annually. Using 6-month intervals, the coupon rate is 3.5% and the yield is 3%.

t	CF_t	$\text{PV}(CF_t)$	$t \cdot \text{PV}(CF_t)$
1	3.5	3.40	3.40
2	3.5	3.30	6.60
3	3.5	3.20	9.60
4	3.5	3.11	12.44
5	3.5	3.02	15.10
6	3.5	2.93	17.59
7	3.5	2.85	19.92
8	103.5	81.70	653.63
		103.50	738.28

Measures of Interest-Rate Risk

Example (cont):

t	CF_t	$\text{PV}(CF_t)$	$t \cdot \text{PV}(CF_t)$
1	3.5	3.40	3.40
2	3.5	3.30	6.60
3	3.5	3.20	9.60
4	3.5	3.11	12.44
5	3.5	3.02	15.10
6	3.5	2.93	17.59
7	3.5	2.85	19.92
8	103.5	81.70	653.63
		103.50	738.28

- Duration (in 1/2 year units) is

$$D = (738.28)/103.50 = 7.13$$

- Modified duration (volatility) is

$$D^* = D/(1+y) = 7.13/1.03 = 6.92$$

- Price risk at y=0.03 is

$$\Delta = D^* \times P = (6.92)(103.5) = 716$$

- **Note: If the yield moves up by 0.1%, the bond price decreases by 0.6860%**

Measures of Interest-Rate Risk

Macaulay Duration

- Duration decreases with coupon rate
- Duration decreases with YTM
- Duration usually increases with maturity
  - For bonds selling at par or at a premium, duration always increases with maturity
  - For deep discount bonds, duration can decrease with maturity
  - Empirically, duration usually increases with maturity

Measures of Interest-Rate Risk

Macaulay Duration

- For intra-year coupons and annual yield *y*

$$\text{Annual} \quad D_m = \sum_{k=1}^{T} k \cdot \omega_k / q$$

$$\text{Annual} \quad D_m^* = \text{Annual} \ D_m / (1 + \frac{y}{q})$$

Convexity

- Sensitivity of duration to yield changes

$$\frac{\partial^2 P}{\partial y^2} = \frac{1}{(1+y)^2} \sum_{k=1}^{T} k \cdot (k+1) \cdot \frac{C_k}{(1+y)^k}$$

$$\frac{1}{P} \frac{\partial^2 P}{\partial y^2} = V_m$$

Measures of Interest-Rate Risk

- Relation between duration and convexity:

$$
\begin{aligned}
P(y') &\approx P(y) + \frac{\partial P}{\partial y}(y) \cdot (y' - y) + \frac{\partial^2 P}{\partial y^2}(y) \cdot \frac{(y'-y)^2}{2} \\
&= P(y) \cdot \left[\ 1 - D_m^*(y'-y) + \frac{1}{2} V_m (y'-y)^2\ \right]
\end{aligned}
$$

- Second-order approximation to bond-price function
- Portfolio versions:

$$
\begin{aligned}
P &= \sum_j P_j \\
D_m^*(P) &\equiv -\frac{1}{P}\frac{\partial P}{\partial y} = \sum_j \frac{P_j}{P} D_{m,j}^* \\
V_m^*(P) &\equiv -\frac{1}{P}\frac{\partial^2 P}{\partial y^2} = \sum_j \frac{P_j}{P} V_{m,j}^*
\end{aligned}
$$

Measures of Interest-Rate Risk

Numerical Example For Duration and Convexity
6% 4-Year Bond, Yield-to-Maturity = 6%, P=100

k	C_k	$\frac{C_k}{(1+\frac{0.06}{2})^k}$	$\frac{kC_k}{2P(1+\frac{0.06}{2})^k}$	$\frac{k(k+1)C_k}{4P(1+\frac{0.06}{2})^k}$
1	3	2.912621	0.014563	0.014563
2	3	2.827787	0.028277	0.042416
3	3	2.745424	0.041181	0.082362
4	3	2.665461	0.053309	0.133273
5	3	2.587826	0.064695	0.194086
6	3	2.512452	0.075373	0.263807
7	3	2.439274	0.085374	0.341498
8	103	81.309151	3.252366	14.635647

Measures of Interest-Rate Risk

$$
\begin{aligned}
D_m^* &= \frac{1}{1+\frac{0.06}{2}} \sum_{k=1}^{8} \frac{kC_k}{2P(1+\frac{0.06}{2})^k} = 3.509846 \\
V_m &= \frac{1}{(1+\frac{0.06}{2})^2} \sum_{k=1}^{8} \frac{k(k+1)C_k}{4P(1+\frac{0.06}{2})^k} = 14.805972 \\
P(y') &\approx P(0.06)\left(1 - 3.509846(y'-0.06) + \right. \\
&\qquad \left. 14.805972\frac{(y'-0.06)^2}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
P(0.08) &\approx P(0.06)(1-0.0701969+0.0029611) \\
&\approx 93.276427
\end{aligned}
$$

$$
P(0.08) = 93.267255
$$

Corporate Bonds and Default Risk

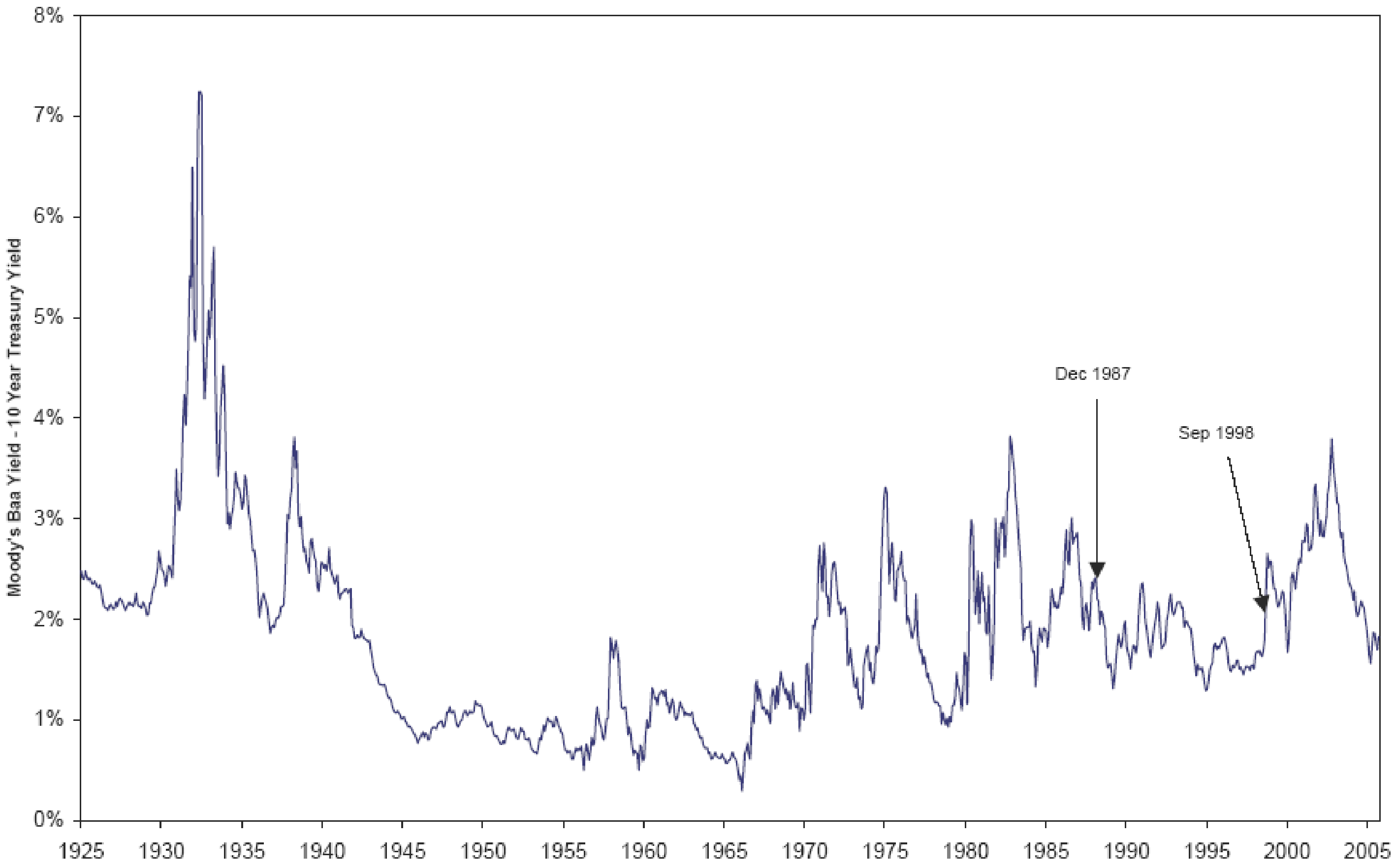
Non-Government Bonds Carry Default Risk

- A **default** is when a debt issuer fails to make a promised payment (interest or principal)
- Credit ratings by rating agencies (e.g., Moody's and S&P) provide indications of the likelihood of default by each issuer.

Credit Risk	Moody's	S&P	Fitch
Investment Grade			
Highest Quality	Aaa	AAA	AAA
High Quality (Very Strong)	Aa	AA	AA
Upper Medium Grade (Strong)	A	A	A
Medium Grade	Baa	BBB	BBB
Not Investment Grade			
Somewhat Speculative	Ba	BB	BB
Speculative	B	B	B
Highly Speculative	Caa	CCC	CCC
Most Speculative	Ca	CC	CC
Imminent Default	C	C	C
Default	C	D	D

Corporate Bonds and Default Risk

Moody's Baa ˜ 10-Year Treasury Yield



Source: Fung and Hsieh (2007)

What's In The Premium?

- Expected default loss, tax premium, systematic risk premium (Elton, et al 2001)
  - 17.8% contribution from default on 10-year A-rated industrials
- Default, recovery, tax, jumps, liquidity, and market factors (Delianedis and Geske, 2001)
  - 5-22% contribution from default
- Credit risk, illiquidity, call and conversion features, asymmetric tax treatments of corporates and Treasuries (Huang and Huang 2002)
  - 20-30% contribution from credit risk
- Liquidity premium, carrying costs, taxes, or simply pricing errors (Saunders and Allen 2002)

Corporate Bonds and Default Risk

Decomposition of Corporate Bond Yields

- **Promised YTM** is the yield if default does not occur
- **Expected YTM** is the probability-weighted average of all possible yields
- **Default premium** is the difference between promised yield and expected yield
- **Risk premium** (of a bond) is the difference between the expected yield on a risky bond and the yield on a risk-free bond of similar maturity and coupon rate

Example: Suppose all bonds have par value $1,000 and

- 10-year Treasury STRIPS is selling at $463.19, yielding 8%
- 10-year zero issued by XYZ Inc. is selling at $321.97
- Expected payoff from XYZ's 10-year zero is $762.22

Corporate Bonds and Default Risk

- For the 10-year zero issued by XYZ:

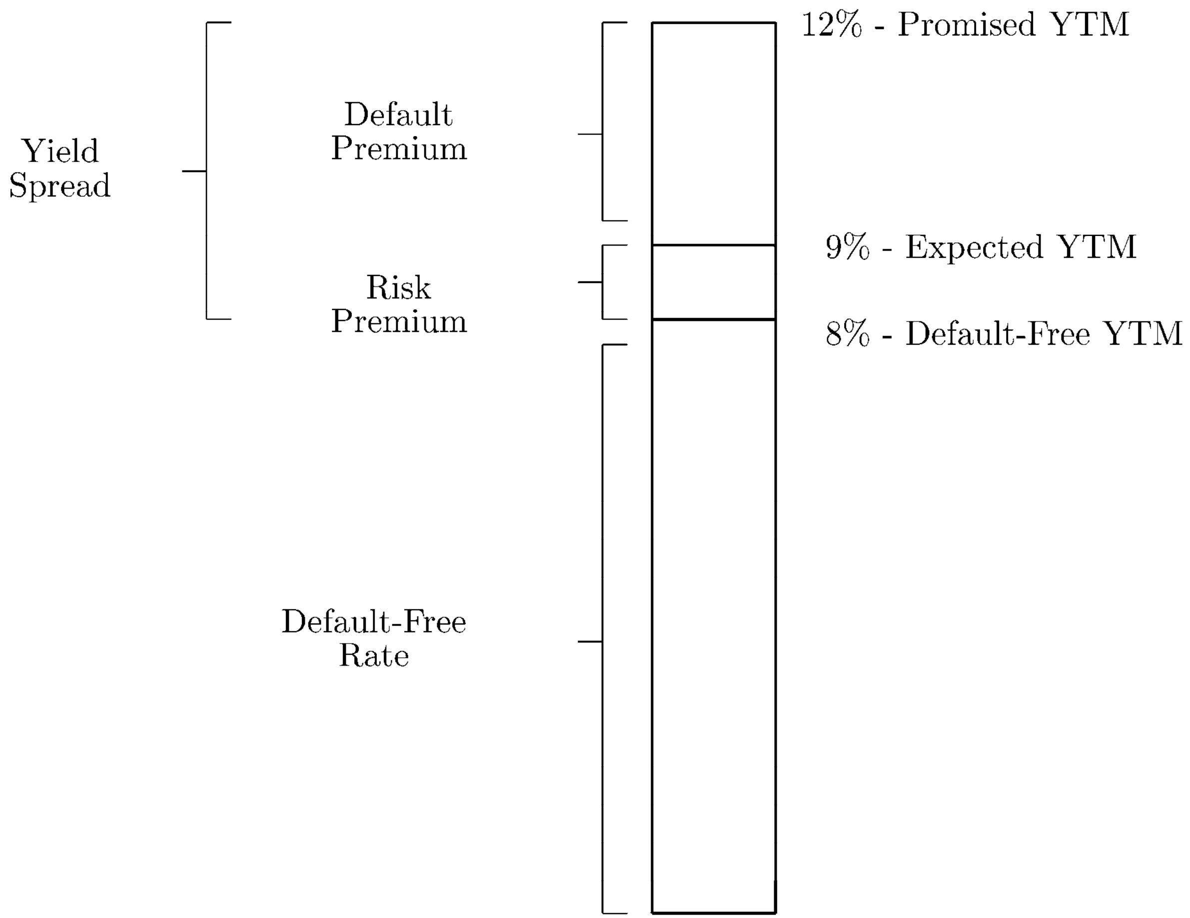
$$\text{Promised YTM} = \left(\frac{1000.00}{321.97}\right)^{1/10} - 1 = 12\%$$

$$\text{Expected YTM} = \left(\frac{762.22}{321.97}\right)^{1/10} - 1 = 9\%$$

$$\begin{aligned}\text{Default Premium} &= \text{Promised YTM} - \text{Expected YTM} \\ &= 12\% - 9\% = 3\%\end{aligned}$$

$$\begin{aligned}\text{Risk Premium} &= \text{Expected YTM} - \text{Default-free YTM} \\ &= 9\% - 8\% = 1\%\end{aligned}$$

Decomposition of Corporate Bond Yields



Why Securitize Loans?

- Repack risks to yield more homogeneity within categories
- More efficient allocation of risk
- Creates more risk-bearing capacity
- Provides greater transparency
- Supports economic growth
- Benefits of sub-prime market

But Successful Securitization Requires:

- Diversification
- Accurate risk measurement
- “Normal” market conditions
- Reasonably sophisticated investors

"Confessions of a Risk Manager" in *The Economist*, August 7, 2008:

Like most banks we owned a portfolio of different tranches of collateralised-debt obligations (CDOs), which are packages of asset-backed securities. Our business and risk strategy was to buy pools of assets, mainly bonds; warehouse them on our own balance-sheet and structure them into CDOs; and finally distribute them to end investors. **We were most eager to sell the non-investment-grade tranches, and our risk approvals were conditional on reducing these to zero.** We would allow positions of the top-rated AAA and super-senior (even better than AAA) tranches to be held on our own balance-sheet as the default risk was deemed to be well protected by all the lower tranches, which would have to absorb any prior losses.

The Sub-Prime Crisis

"Confessions of a Risk Manager" in *The Economist*, August 7, 2008:

In May 2005 we held AAA tranches, expecting them to rise in value, and sold non-investment-grade tranches, expecting them to go down. From a risk-management point of view, this was perfect: have a long position in the low-risk asset, and a short one in the higher-risk one. **But the reverse happened of what we had expected: AAA tranches went down in price and non-investment-grade tranches went up, resulting in losses as we marked the positions to market.**

This was entirely counter-intuitive. Explanations of why this had happened were confusing and focused on complicated cross-correlations between tranches. In essence it turned out that there had been a short squeeze in non-investment-grade tranches, driving their prices up, and a general selling of all more senior structured tranches, even the very best AAA ones.

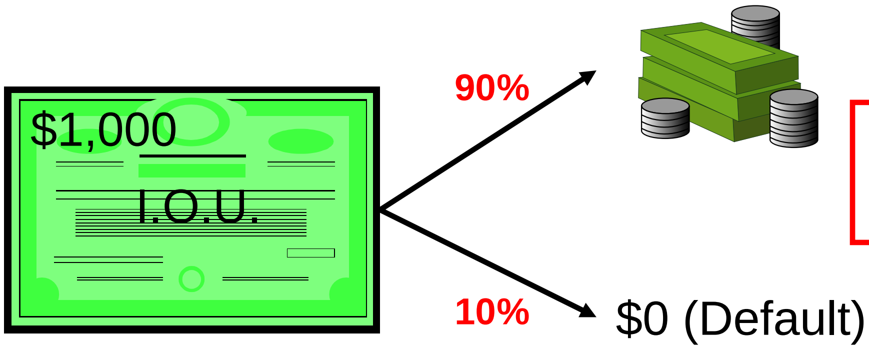
An Illustrative Example

Consider Simple Securitization Example:

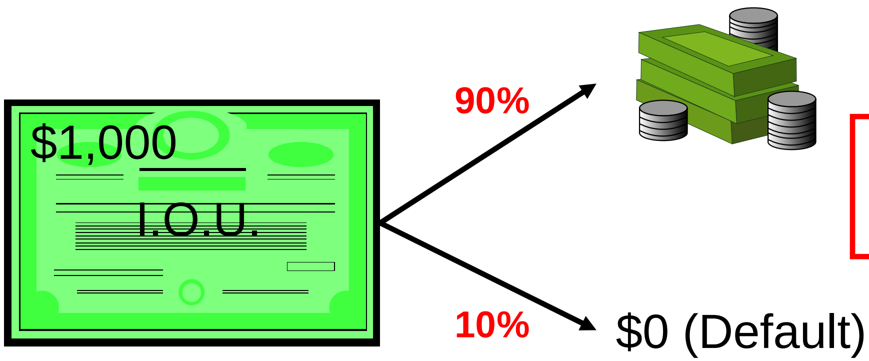
- Two identical one-period loans, face value $1,000
- Loans are risky; they can default with prob. 10%
- Consider packing them into a portfolio
- Issue two new claims on this portfolio, S and J
- Let S have different (higher) priority than J
- What are the properties of S and J?
- What have we accomplished with this “innovation”?

Let’s Look At The Numbers!

An Illustrative Example

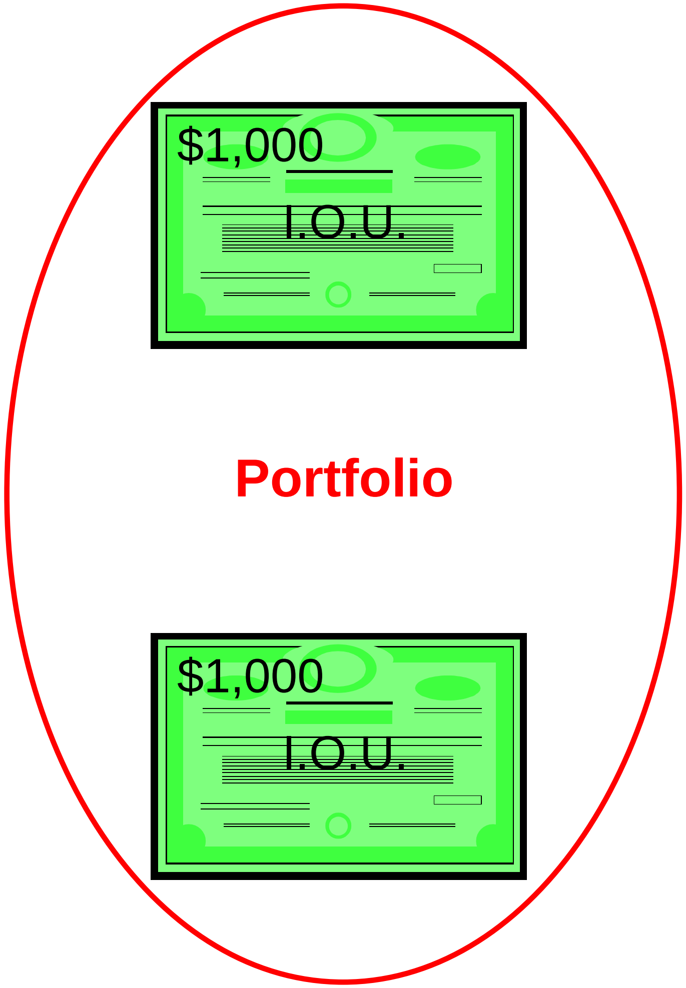


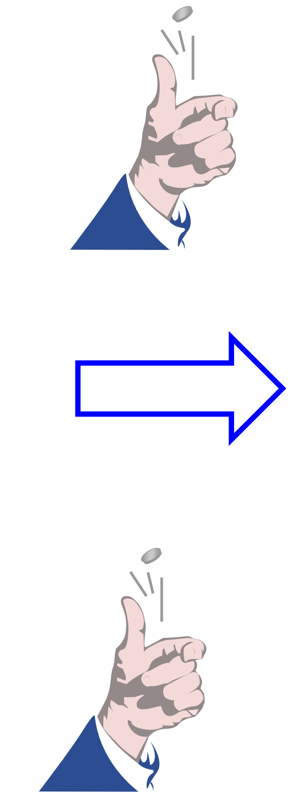
Price = 90% × $1,000 + 10% × $0
= $900



Price = 90% × $1,000 + 10% × $0
= $900

An Illustrative Example

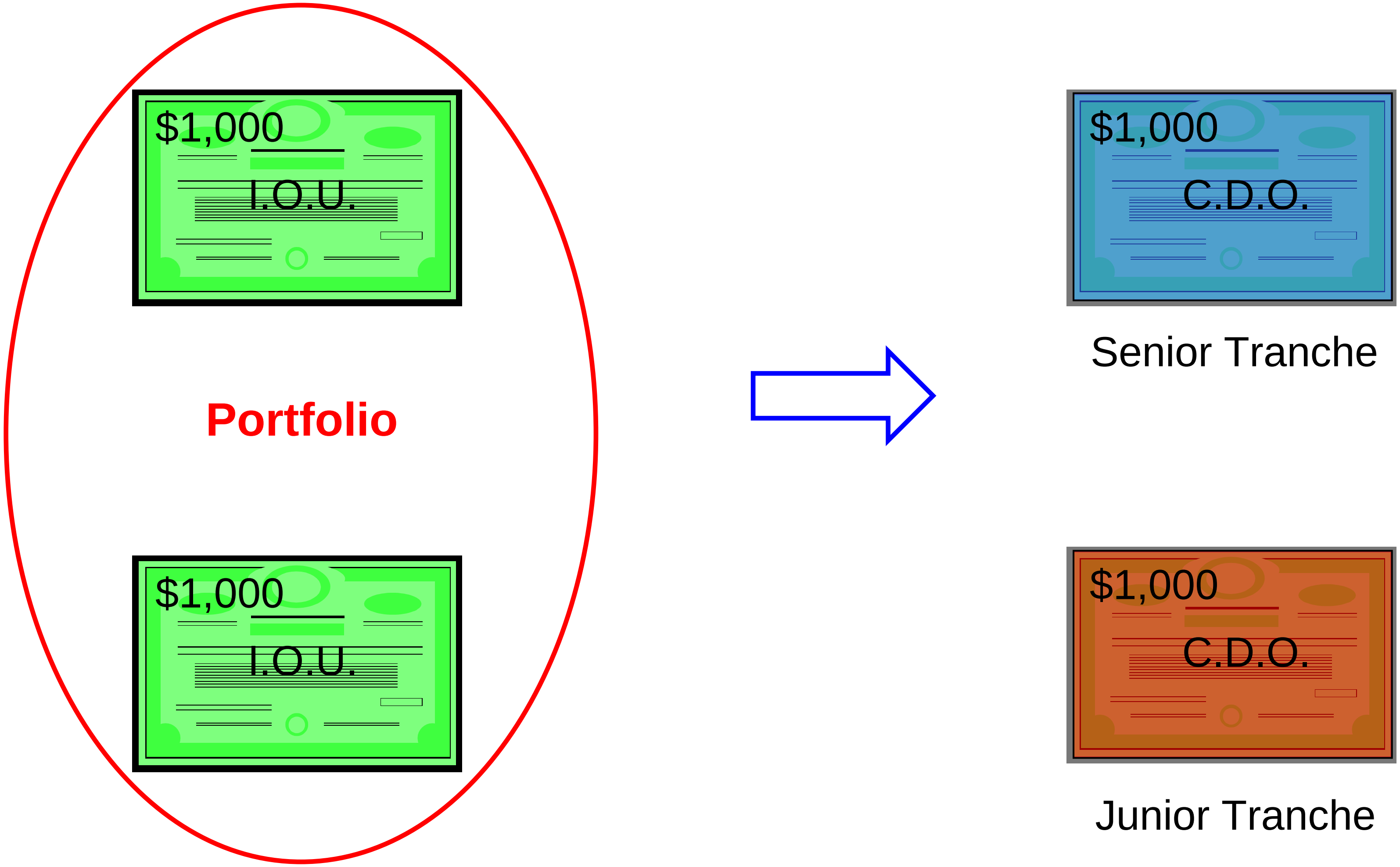




Assuming Independent Defaults

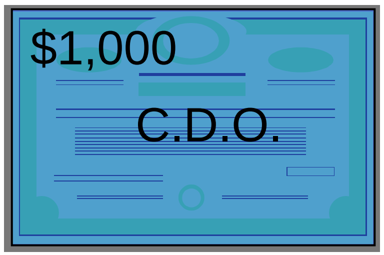
Portfolio Value	Prob.
$2,000	81%
$1,000	18%
$0	1%

An Illustrative Example

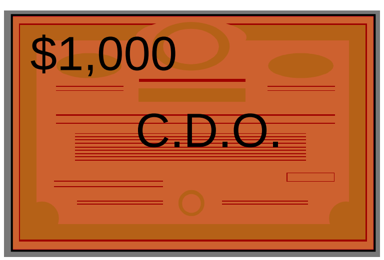


An Illustrative Example

Assuming Independent Defaults



Senior Tranche



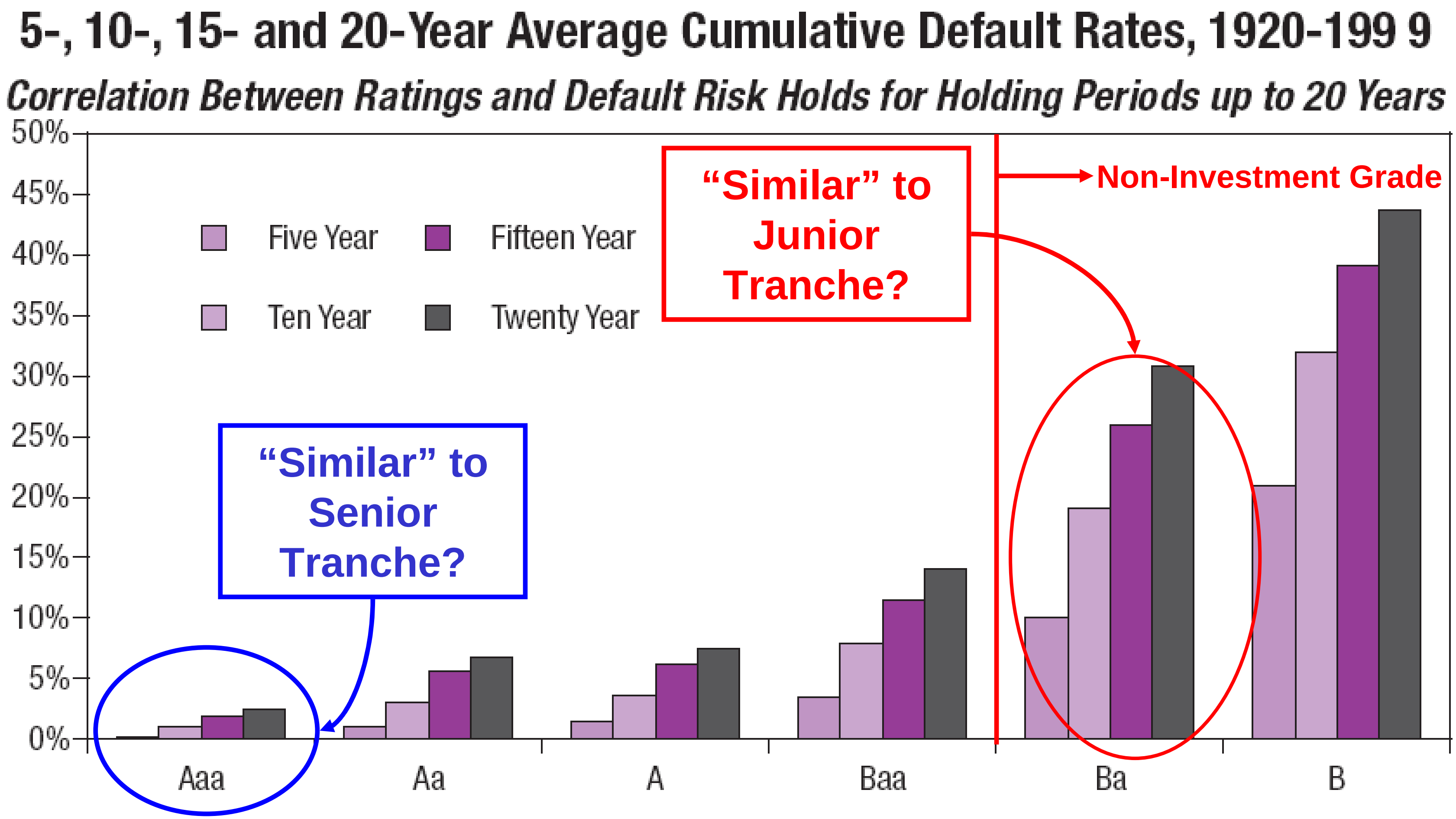
Junior Tranche

Portfolio Value	Prob.	Senior Tranche	Junior Tranche
$2,000	81%	$1,000	$1,000
$1,000	18%	$1,000	$0
$0	1%	$0	$0

Bad State For Senior Tranche (1%)

Bad State For Junior Tranche (19%)

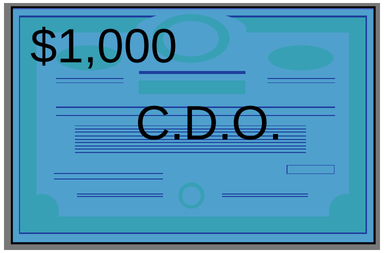
An Illustrative Example



Source: Moody's

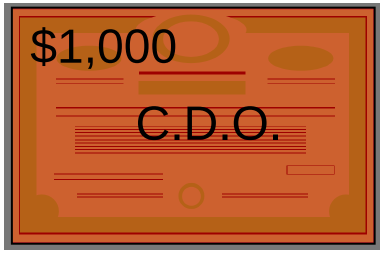
An Illustrative Example

Assuming Independent Defaults



Senior Tranche

Portfolio Value	Prob.	Senior Tranche	Junior Tranche
$2,000	81%	$1,000	$1,000
$1,000	18%	$1,000	$0
$0	1%	$0	$0

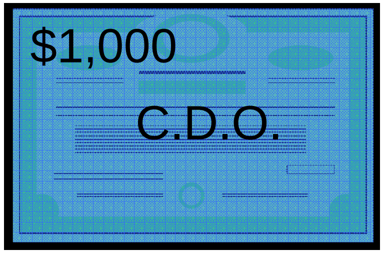


Junior Tranche

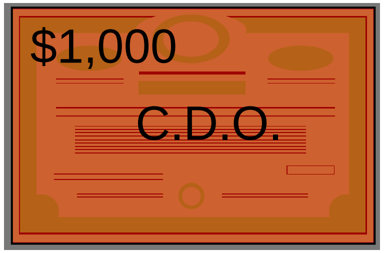
Price for Senior Tranche = 99% × $1,000 + 1% × $0
= $990

Price for Junior Tranche = 81% × $1,000 + 19% × $0
= $810

An Illustrative Example



Senior Tranche



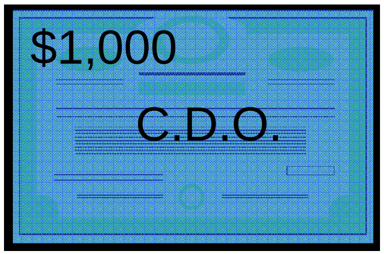
Junior Tranche

Assuming Independent Defaults

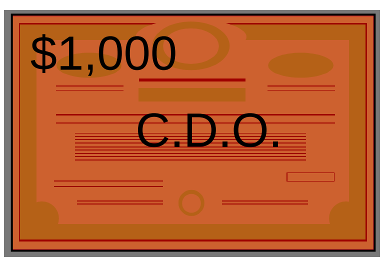
Portfolio Value	Prob.	Senior Tranche	Junior Tranche
$2,000	81%	$1,000	$1,000
$1,000	18%	$1,000	$0
$0	1%	$0	$0

But What If Defaults Become Highly Correlated?

An Illustrative Example



Senior Tranche



Junior Tranche

Assuming Perfectly Correlated Defaults

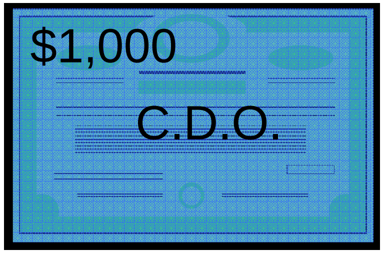
Portfolio Value	Prob.	Senior Tranche	Junior Tranche
$2,000	90%	$1,000	$1,000
$0	10%	$0	$0

Bad State For Senior Tranche (10%)

Bad State For Junior Tranche (10%)

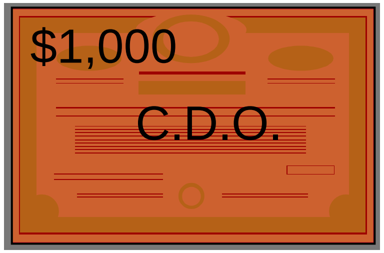
An Illustrative Example

Assuming Perfectly Correlated Defaults



Senior Tranche

Portfolio Value	Prob.	Senior Tranche	Junior Tranche
$2,000	90%	$1,000	$1,000
$0	10%	$0	$0



Junior Tranche

Price for Senior Tranche $= 90\% \times \$1{,}000 + 10\% \times \$0$
$= \$900$ (was \$990)

Price for Junior Tranche $= 90\% \times \$1{,}000 + 10\% \times \$0$
$= \$900$ (was \$810)

Implications

To This Basic Story, Add:

- Very low default rates (new securities)
- Very low correlation of defaults (initially)
- Aaa for senior tranche (almost riskless)
- Demand for senior tranche (pension funds)
- Demand for junior tranche (hedge funds)
- Fees for origination, rating, leverage, etc.
- Insurance (monoline, CDS, etc.)
- Equity bear market, FANNIE, FREDDIE

Then, National Real-Estate Market Declines

- Default correlation rises
- Senior tranche declines
- Junior tranche increases
- Ratings decline
- Unwind ⇒ Losses ⇒ Unwind ⇒ …

Key Points

- Valuation of riskless pure discount bonds using NPV tools
- Coupon bonds can be priced from discount bonds via arbitrage
- Current bond prices contain information about future interest rates
- Spot rates, forward rates, yield-to-maturity, yield curve
- Interest-rate risk can be measured by duration and convexity
- Corporate bonds contain other sources of risk

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